

### Practice Questions for Midterm 2 - Math 1060Q - Fall 2013

The following is a selection of problems to help prepare you for the second midterm exam. Please note the following:

- there may be mistakes – email [steven.pon@uconn.edu](mailto:steven.pon@uconn.edu) if you find one.
- the distribution of problems on this review sheet does not reflect the distribution that will be on the exam.
- learning math is about more than just memorizing the steps to certain problems. You have to understand the concepts behind the math. In order to test your understanding of the math, you may see questions on the exam that are unfamiliar, but that rely on the concepts you've learned. Therefore, you should concentrate on learning the underlying theory, not on memorizing steps without knowing *why* you're doing each step. (Some examples of “unfamiliar” problems can be found in the sample problems below, to give you an idea.)

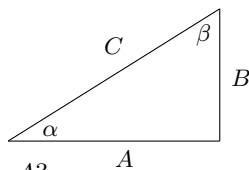
**Now trig functions. Angles can be measured in degrees or radians, and though we primarily use radians, it's worth knowing how to convert. More importantly, you should know the unit circle very well!**

1. Convert  $30^\circ$  to radians.
2. Convert  $\frac{\pi}{2}$  radians to degrees.
3. Which of the following angles are coterminal?  $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{-7\pi}{4}$
4. What is an angle in the interval  $[\pi, 2\pi]$  that corresponds to the angle  $-\frac{\pi}{6}$  (i.e., is coterminal with  $-\frac{\pi}{6}$ )?
5. What is an angle in the interval  $[-\pi, 0]$  that corresponds to the angle  $\frac{5\pi}{3}$  (i.e., is coterminal with  $-\frac{5\pi}{3}$ )?
6. Draw the unit circle. Label the angles  $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{3\pi}{4}, \pi, \frac{7\pi}{6}, \frac{7\pi}{4}, 2\pi, -\frac{\pi}{4}, -\frac{5\pi}{3}$ . Label the coordinates of the points on the unit circle that correspond to those angles.
7. Find an angle in  $[0, 2\pi)$  that is “coterminal” with the angle  $\frac{65\pi}{6}$ .
8. Find an angle in  $[0, 2\pi)$  that is “coterminal” with the angle  $\frac{14\pi}{5}$ .
9. Find the “reference angle” for the angle  $\frac{7\pi}{4}$ . (The reference angle is the angle in  $[0, \frac{\pi}{2}]$  formed between a given angle and the  $x$ -axis, so for example, the reference angle for  $\frac{2\pi}{3}$  is  $\frac{\pi}{3}$ .)
10. Find the “reference angle” for the angle  $\frac{5\pi}{6}$ .
11. Find the “reference angle” for the angle  $\frac{17\pi}{5}$ .

**Do you understand what the trigonometric functions are? How to calculate them, both using triangles or the unit circle?**

12. Define sine, cosine, tangent, cosecant, secant, cotangent. What are their domain and range?
13. What is  $\sin(30^\circ)$ ?
14. What is  $\csc(\frac{5\pi}{6})$ ?
15. If  $t = \frac{20\pi}{3}$ , what are  $\sin(t)$ ,  $\csc(t)$ , and  $\cot(t)$ ?
16. If  $\cot(t) = 1$  and  $t$  is in the interval  $[\pi, 2\pi]$ , then what is  $\sin(t)$ ?
17. If  $\cos(t) = -\frac{1}{2}$  and  $t$  is in the interval  $[\pi, 2\pi]$ , then what is  $\tan(t)$ ?
18. Given that  $\cos(\theta) = \frac{2}{7}$  and  $\theta$  is in quadrant IV, find  $\sin(\theta)$ .
19. Given that  $\tan(\theta) = -\frac{3}{5}$  and  $\theta$  is in quadrant II, find  $\csc(\theta)$ .

20. Use the triangle below to answer these questions.



- (a) Say  $\alpha = \frac{\pi}{4}$  and  $B = 8$ . What is  $A$ ?
- (b) Say  $\beta = \frac{\pi}{6}$  and  $A = 20$ . What is  $C$ ?
- (c) Say  $A = 5$  and  $B = 10$ . What is  $\alpha$ ?
21. Is  $\sin(\frac{19\pi}{20})$  closest to -1, 0, or 1?

**What about going “backwards?” If you know the values of trig functions, can you work backwards to find out what the angle might be?**

22. If  $\sin(t) = \frac{\sqrt{2}}{2}$ , what might  $t$  be? Give all possible solutions in  $[-\pi, \pi]$ .
23. If  $\sin t = 1$ , what might  $t$  be? Give all possible solutions in  $[0, 2\pi]$ .
24. If  $\cos t = \frac{1}{2}$ , what might  $t$  be? Give all possible solutions in  $[0, 2\pi]$ .
25. If  $\tan t = -1$ , what might  $t$  be? Give all possible solutions in  $[0, 2\pi]$ .
26. Find all values of  $t$  in the interval  $[0, 2\pi]$  such that  $\sin t = 0$ .
27. Find all values of  $t$  in the interval  $[0, 2\pi]$  such that  $\csc t = 2$ .
28. Find all values of  $t$  in the interval  $[0, 2\pi)$  such that  $\cot t = \sqrt{3}$ .

**Like any other function, we sometimes want to graph trigonometric functions. Can you?**

29. Graph  $\sin(x)$ ,  $\cos(x)$ , and  $\tan(x)$ .
30. Graph  $f(x) = -3\sin(x)$ .
31. Graph  $g(x) = \cos(x - \frac{\pi}{2})$ . Can you find another function that has this same graph?
32. Graph  $h(x) = 2\sin(4x)$ .
33. Where does  $\csc(x)$  have a vertical asymptote?
34. Graph  $2\sec(2x)$ .
35. Graph  $5\sin(\frac{x}{3} - \pi) + .5$ .
36. Graph  $\cos(2(x + \frac{\pi}{4}))$ .

**Can you solve equations that involve trigonometric functions?**

37. Find all solutions to  $2\sin(x) + 1 = 0$  in  $[0, 2\pi)$ .
38. Find all solutions to  $3\cos x = 3$ .
39. Find all solutions to  $3\sin x - 4 = \sin x - 2$  in  $[0, 2\pi)$ .
40. Find all solutions to  $4\cos^2 x - 1 = 0$  in  $[0, 2\pi)$ .
41. Find all solutions to  $\cos(2x) = \frac{1}{2}$  in  $[0, 2\pi)$ .
42. Find all solutions to  $\tan^2(x) = 3$ .
43. Find all solutions to  $\sin x + \sqrt{2} = -\sin x$  in  $[0, 2\pi)$ .

44. Find all solutions to  $2 \cos(3x - 1) = 0$  in  $[0, 2\pi)$ .
45. Find all solutions to  $\sec x = 2 \cos x$  in  $[0, 2\pi)$ .
46. Find all solutions to  $2 \sin^2 x - \sin x - 1 = 0$  in  $[0, 2\pi)$ .
47. Find all solutions to  $\sin x \cos x + \cos x = 0$  in  $[0, 2\pi)$ .
48. Find all solutions to  $\sin(2x) = -\cos(2x)$ .

**Sometimes you need to use a trigonometric identity to help solve an equation. You'll need to use trig identities in the following problems.**

49. Find all solutions to  $2 \cos^2 x + 3 \sin x = 3$  in  $[0, 2\pi)$ .
50. Find all solutions to  $\sin(2x) = \sqrt{3} \cos x$  in  $[0, 2\pi)$ .
51. Find all solutions to  $2\sqrt{2} \sin(2t) - 2 \tan(2t) = 0$  in  $[0, 2\pi)$ .
52. Find all solutions to  $2 \cot^2 x + \csc^2 x - 2 = 0$  in  $[0, 2\pi)$ .
53. Find all solutions to  $\cos(2x) = \cos x$  in  $[0, 2\pi]$ .

**Just like with other functions we've studied, trigonometric functions have inverses. Well, partial inverses (since trig functions aren't one-to-one). Do you know what the inverse trig functions are? Do you know their domain and range? Can you compute them for some values? Can you graph them?**

54. What is  $\arccos \frac{1}{2}$ ?
55. What is  $\arcsin \frac{-\sqrt{2}}{2}$ ?
56. What is  $\arctan 0$ ?
57. What is  $\sin^{-1}(\sin(\frac{3\pi}{2}))$ ?
58. What is  $\tan(\arctan(-3))$ ?
59. What is  $\arccos(\cos(-\frac{4\pi}{9}))$ ?
60. What is  $\tan(\sin^{-1}(\frac{\sqrt{2}}{2}))$ ?
61. What is  $\tan(\arccos(\frac{5}{13}))$ ?
62. Express a solution to the equation  $\tan(x - 3) = 5$  using inverse trigonometric functions.
63. Express a solution to the equation  $\sin(4x) + 1 = \frac{2}{3}$  using inverse trigonometric functions.
64. Graph the function  $\cos^{-1}(2x)$ .
65. Graph the function  $\arctan(x) - \frac{\pi}{2}$ .
66. Graph the functions  $\csc^{-1}(x)$  and  $\frac{1}{\csc(x)}$ .

**Can you apply your knowledge to real-world applications?**

67. How long a ladder do you need if you want to reach a window that's 20 feet off the ground? You're on uneven terrain, so leaning a ladder any steeper than 60 degrees is unsafe.
68. An equilateral triangle is circumscribed around a circle of radius 5 (thus, the center of the triangle and the center of the circle are at the same point). What is the area of the triangle?
69. A rope hanging straight down from a pole is 4 feet longer than the pole (that is, the rope hangs straight down and there's 4 feet of extra rope lying on the ground). When the rope is picked up and stretched taut, it hits the ground 8 feet away from the pole. How tall is the pole?

70. The number of hours of daylight in a day can be modeled as a sinusoidal curve. Assume that the longest day of the year is June 21, which has 15.2 hours of daylight, and the shortest day of the year is December 21, which has 9.1 hours of daylight (this is close to the real numbers in CT). Let's start the clock on January 1, and say Jan 1 corresponds to  $t = 0$ . Find a sinusoid to model the number of hours of daylight as a function of the day of the year. Use this to estimate the number of hours of daylight there will be tomorrow. The numbers in this problem are messy, so it's not a great exam problem...but it's kind of fun.
71. A 27 inch (diameter) bicycle wheel turns at 120 revolutions per minute. There is a nail in the wheel. Find a function to model the height of the nail above the ground at any point in time, assuming we start the clock when the nail hits the pavement.
72. A weight hangs at the end of a spring, causing it to bounce up and down. At its lowest point, the weight sits 60 cm above the ground, and at its highest point, it reaches 100 cm above the ground. Every 10 seconds, the weight bounces up and down 4 times. The height of the weight above the ground can be modeled by a sine curve. Supposing the weight starts from its lowest point at time  $t = 0$  seconds, find a sinusoid to model the height of the weight at time  $t$ .
73. A plane is flying at an altitude of 36,000 feet. Looking down, the pilot sees a ship at an angle of depression of 30 degrees. He sees a submarine at an angle of depression of 60 degrees in the same direction. How far apart are the ship and submarine?

**Here are a few additional problems that didn't make it into any of the above sections.**

74. Calculate  $\sin(\frac{\pi}{3} - \frac{\pi}{4})$  without using a calculator.
75. Simplify the expression using identities as needed:  $(1 - 2\cos^2 x + \cos^4 x)$ .
76. Calculate  $\cos(\frac{7\pi}{8})$  without using a calculator.
77. Simplify  $\frac{3}{\tan^2 x + 1}$ .
78. True or false?  $3\cos x + 3\sin x \tan x = 3\sec x$ .
79. True or false?  $\frac{3\cot^3 x}{\csc x} = 3\cos x(\csc^2 x - 1)$
80. Show work to prove that  $\frac{3\sec\theta - 3}{1 - \cos\theta} = 3\sec\theta$ .